APPLICATION OF A FINITE ELEMENT MODEL UPDATING METHODOLOGY ON THE IPEX-II STRUCTURE

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ABSTRACT. *This study presents a methodology for updating the finite element model of a structure using an incomplete set of experimentally obtained modal frequencies and modeshapes. The model updating problem involves the least squares minimization of the modal dynamic force balance residuals subject to quadratic inequality constraints. The measure of fit and constraints properly account for the measurement and modeling errors in the model updating. The finite element model of the Interferometry Program Experiment IPEX-II boom structure is updated using modal tests obtained from pre-flight modal experiments. The sensitivity of the updated model to the structural parameterization (model substructuring) and the number and type of measured modes considered in the model updating methodology is explored. Significant improvements in the updated models are observed for all cases considered. The adequacy of the chosen parameterized class of finite element models of the IPEX-II structure is also determined.*

NOMENCLATURE

K	Stiffness matrix
M	Mass matrix
P	Matrix of zeroes and ones
R	Weighting matrix
ϕ	Modeshape expansion vector
$\hat{\phi}$	Measured modeshape vector
θ	vector of model parameters
a	Measured degrees of freedom
$\hat{\omega}$	Measured modal frequency

1. INTRODUCTION

The need for model updating arises in the process of constructing a theoretical model of a structure. In order to improve the accuracy in the model response predictions, the pre-test finite element model of a structure is updated to match available dynamic test data. The general problem of structural model updating involves the selection of the model from a parameterized class of models that provides the best fit to the measured dynamic data as judged by an appropriately selected measure of fit. The parameters involved in the updating are structural stiffness and mass properties, including boundary conditions as well as fixity conditions at the structural joints. The following are the difficulties associated with this inverse problem: 1) the chosen class of parametric finite element models is not representative of the actual structural behavior for all possible values of the model parameters; 2) the measured dynamic data are contaminated by measurement error; 3) the set of observed DOF is usually a small subset of the set of model DOF due to the limited number of sensors used or due to limited accessibility within a structure; and 4) the number of identifiable modes of vibration is much less than the number of model DOF due to large measurement noise for higher modes, limited bandwidth in the response and hardware limitations. As a result, structural model updating is a challenging problem which may also lead to non-unique solutions and ill-conditioning [1-4].

In past years, several studies have been devoted in reconciling finite element models with measured time history or modal data (e.g. [1,3,5-13]) Each method has its own advantages and shortcomings and there is no acceptable methodology for successfully treating the model updating problem. A model updating method should account for measurement and modeling errors, as well as properly incorporate available prior information about the possible range of variation of the system parameters. Model updating methods based on Bayesian statistical inference show great promise for properly accounting for measurement and modeling errors, as well as properly addressing many of the difficulties encountered in the model updating problem, especially those associated with non-uniqueness and model (response) prediction accuracy [2,6,13,14].

A modal-based model updating methodology was recently developed [11,12] that combines available mode-shape expansion techniques [15,16] with updating capabilities for predicting both the location and size of errors in the pre-test finite element model of a structure. Other model updating methodologies based on various mode-shape expansion can be found in references [9,10,14]. Applications of these methodologies were focused on structural damage detection and structural health monitoring. These techniques, which use the modeshape components as unknowns to be determined by the data, have the advantage of avoiding the problem of identifying the correspondence between model and measured modes. Moreover, the computation of modal frequencies and modeshapes of the finite element model is avoided. Also, the expanded modeshapes can be used for element strain energy measures to predict potential damage locations or locations of errors in the properties of the finite element model [11,15].

In this study, the model updating methodology proposed in [11] is first outlined and then applied to update the pre-test finite element model of the IPEX-II structure. The best model out of a chosen class of the finite element models is obtained by employing different parameterizations and by considering different number of modes in the model correlation studies. Based on the updated models obtained for the different cases, recommendations are offered as to the source of the differences and the action to be taken for improving further the quality of the model and the reliability of the model response predictions.

2. MODEL UPDATING METHODOLOGY

A class of linear structural models is used with global mass and stiffness matrices $M(\theta)$ and $K(\theta)$ assembled from the element (or substructure) mass and stiffness matrices, respectively, using a finite element analysis. The set θ includes the uncertain parameters of the model to be assigned values during the search for the optimal model. The parameter set θ may represent mass and stiffness properties at the element or substructure level. Examples of finite element properties that can be included in the parameter set θ are: modulus of elasticity, cross-sectional area, thickness, moment of inertia and mass density of the finite elements comprising the model, as well as spring (translational or rotational) stiffnesses used to model fixity conditions at joints or boundaries. For convenience, the parameterization is chosen such that the pre-test finite element model of the structure corresponds to $\theta = 1$.

Mathematically, the model updating problem is stated as a constrained minimization problem [11,12]:

$$\min_{\theta,\phi_i} \sum_{i=1}^{m} \beta_i \left\| (K(\theta) - \hat{\omega}_i^2 M(\theta))\phi_i \right\|_{R_i}$$

subject to

$$\left\| \hat{\phi}_{ai} - P\phi_i \right\|^2 \leq \alpha_i \left\| \hat{\phi}_{ai} \right\|^2, \quad i = 1, \ldots, m \qquad (2)$$

where $\|(K(\theta) - \hat{\omega}_i^2 M(\theta))\phi_i\|$ is the modal dynamic force balance residuals, m is the number of measured modes, $\hat{\omega}_i$ and $\hat{\phi}_{ai}$ are the experimentally obtained i-th modal frequency and modeshape of the structure at the measured model degrees of freedom a, $\|x\|^2 = x^T x$, $\|x\|_R = x^T R x$, R_i is an appropriately selected weighting matrix which scales the contribution of each mode in the measure of fit (1), and P is a constant matrix of zeroes and ones such that $\phi_{ai} = P\phi_i$.

The proposed inequality constraints (2) provides flexibility in improving the fit between model and measured modal data over the space of the parameter set θ by accounting for the expected measurement error in the mode-shape components, with α_i controlling the expected magnitude of these errors. For example, the extreme values of $\alpha_i = 0$ and $\alpha_i = \infty$ correspond to the cases of perfectly reliable and completely unreliable modeshapes, respectively. In particular, in the case $\alpha_i = \infty$ the modeshape measurements for the ith mode are ignored in the formulation.

The weights R_i are selected to make the i-th modal term $\|(K(\theta) - \hat{\omega}_i^2 M(\theta))\phi_i\|$ in the overall measure of fit (1) non-dimensional. The preferred choice is [11]

$$R_i = K^{-1}(\theta) M(\theta) K^{-1}(\theta) \qquad (3)$$

Under the assumption of perfectly correlated expanded and model mode-shapes, the modal measure becomes proportional to the fractional difference between the squares of the model and measured modal frequencies for mode i, weighted by the scalar β_i [14]. This equivalence provides insight into the problem of specifying the weights β_i. Specifically, from a Bayesian statistical point of view, the weights β_i in (1) and α_i in (2) reflect the magnitude of the measurement errors expected 1)('IM'('('li the experimental and modal frequencies and modeshapes for each mode, respectively. The size of these errors can be computed from a statistical analysis of measurement data taken from multiple modal test analyses.

The unknown quantities involved in the proposed measure of fit (1) include, in addition to the model parameters θ, the components of the vector ϕ_i of the contributing modes at both measured and unmeasured model degrees of freedom. The optimal vector ϕ_i, $i = 1, \ldots, m$ resulting from the minimization is the expanded modeshapes which are consistent with the measured modal data and the updated model.

The optimization in (1) and (2) can be performed using available inequality constaint optimization techniques. However, this is a complex and time-consuming nonlinear optimization problem. A more convenient two-step iterative procedure is used which avoids some of the computational difficulties arising in the constrained minimization of (1) and (2). The details of this procedure are presented in references [11,12,15]. First, expanded modeshapes are ('0111 1put '(1 by solving the constrained minimization problem given the current model of the structure at the k-th iteration step, corresponding to the current optimal value of the parameter set θ designated by $\hat{\theta}^{(k)}$. The robustness and reliability of the modeshape expansion technique for predicting the modeshape ('0111]])[)11('1115 at unmeasured points have 1)('('11 successfully evaluated in a previous study using actual experimental data obtained on the Jet Propulsion Laboratory micro-precision interferometer truss [15-17]. Compared with other modeshape expansion techniques, the least squares minimization technique with quadratic inequality constraints was found 1 0 provide the most reliable mode shape estimates, even in adverse situations of significant measurement and model error. Used with element strain energy measures, it has ill so been demonst rated t () predict significant modeling error locations that could be 11 S(,(1 as a guide in updating the class of finite element models employed in teh model correlation studies [17].

In the second step of the two-step iterative procedure, the parameters of the model are updated using the latest estimates $\varphi_i^{(k+1)}$, $i=1, \ldots, m$ of the complete modeshapes. The optimal values $\hat{\theta}^{(k+1)}$ corresponding to the $k+1$ iteration are obtained by the solution of an unconstrained minimization problem. Since the updated finite element model obtained at the $k+1$ iteration contains inaccuracies (111(' to the fact that the expanded modeshapes are based on an inaccurate model obtained at the previous iteration k, the two step procedure is repeated until convergence is reached. Specifically, the iterative process is terminated when $\theta^{(k+1)} - \theta^{(k)} / \theta^{(k+1)} < tol$ where tol is a user-specified threshold level. It can be shown that the optimal solution θ and ϕ_i obtained from the iterative two-step procedure is also the optimal solution of the original inequality constrained minimization problem described by equations (1) and (2).

3. APPLICATION TO IPEX-II STRUCTURE

The IPEX-II boom structure is a 2.3 m nine-bay three-dimensional ABLE Deployable Articulated Mast (ADAM) with graphite-epoxy mast boom and steel cables supplied by AEC-ABLE, Inc. The ADAM-Mast is constructed from graphite-epoxy longerons and battens, 15-5 1'11 stainless steel 110(1(% and latch assembles, and Cress 302 wire rope. The weight of the IPEX-II boom structure is approximately 31 pounds. Details of the structural connection at the main nodes are shown in Figure 1. The structure is schematically shown in Figure 2.

In the modal test configuration, the ADAM-Mast bare truss was considered in its cantilever position with the fixed end simulated by attaching the four mast/strut interface points of the ADAM-Mast bare truss to a fixed mounting fixture bolted to two tracks on a massive granite table. The granite table was supported to the ground by four wooden blocks. The free end of the mast was attached to a rigid 0.7-in thick aluminum plate of 15.62 pounds. A detailed description of the structure along with the modal test description and setup (i III be found in [18]. A total of 58 accelerometers were used in the modal test '1 he accelerometer locations and orientations are silo\\'11 in Figure 2 and they were selected based on a pre-test analysis and engineering judgement

The class of finite element models of the structure was chosen using engineering experience to represent in sufficient detail the behavior of the connections at the nodes as well as adequately 1110 (1 ('1 some of the local modes expected in the frequency range of interest. Specifically, each of the longeron and the batten members were modelled by seven and four beam elements, respectively. Each diagonal steel cable was 1110(1('1('(1 by four massless beam elements. The polley connection (Figure 1(a)) was 1[10(1('1[('(1 as a single 110(1(" with a lumped mass. The connections at the joints, shown in Figure 1(b), were modelled in detail using plate (J1(lll('tits. Concentrated masses were added to the main nodes of" the till ss to account for the structural mass from the plate and beam elements. The aluminum plate attached to the free end was modelled by five plate elements. The resulting finite element 1[10(1('1 has 5 1 1 1 degrees of freedom.

The geometric nonlinear stiffness due to the large preload on the diagonal steel cables was also considered in the beam elements comprising the finite element model. The stiffness reduction due to the large axial preload in the beam elements was found to be important in the modeling of the correct micro-dynamic behavior of the structure. However, variation of the preload values of approximately 20-30% a nominal value of the 3(K) pounds do not have much effect on the global modes. However they do tend to affect the local bending modes 4 and 5 reported in Table 1.

The finite element model was reduced to 441 degrees of freedom using Guyan reduction [19]. To verify that the reduction does not significantly alter the dynamics of the model in the frequency range of interest, the lowest modal frequencies in the range 10-200Hz of the reduced model were computed and compared to the modal frequencies of the original 5 11 1-DOF model. A very good agreement within the range of 1% was observed for all modal frequencies considered in the model updating and over a wide range of nominal model parameter values. It is also assumed that the structure will behave linearly within the test vibration levels. This assumption was also validated experimentally [18].

Simulated modal data generated from the IPEX-II mini-structure were first used to assess the strengths, limitations, and overall performance of the two-step iterative model updating methodology in relation to the number of measured modes, number and location of sensors, as well as number and type of model parameters. Extensive simulated studies performed with simulated noise-contaminated data have shown that the model updating methodology has acceptable simulation accuracy, and fast convergence. These simulated studies provide valuable insight into the type of model parameters that could be reliably identified from the modal test data.

Comparisons between the nominal analytical model and the test data revealed the necessity for updating the analytical model involving, as parameters, the modulus of elasticity of the longeron members and the stiffness of the steel cable, as reported in [18]. The updated model in [18] is further used in this study as the nominal pre-test model.

The following model parameters are chosen to be updated. For each bay, the model parameters include the axial and bending stiffness of the longeron and the batten members, the axial stiffness of the steel cable, and the concentrated mass modeling the steel cable/pulley connection. Considering the fact that each bay has been constructed to be identical in theory with all nine bays, the values of the six model parameters of any one of the bays are taken to be fully correlated with the corresponding values of all other bays. This limits the number of the model parameters to be updated to a total of six parameters. Extensive preliminary studies with both simulated and test data have revealed that for the range of parameter values considered, the modal prediction results based on the updating model are insensitive to the flexural stiffness of the longeron members and the axial stiffness of the batten members. Therefore, the modulus of elasticity of the longeron and the batten members, designated by θ_1 and θ_3, respectively, were chosen to be updated. The other two parameters included in the model parameter set are the modulus of elasticity of the steel cable and the mass at the pulley connections, designated by θ_2 and θ_4, respectively.

Model updating results were obtained for various substructuring cases and for different number of modes considered in the model updating methodology. Specifically, the cases of three and four model parameters were considered. Moreover, the cases of matching modal data from five, seven, nine and eleven modes were explored separately and model updating results were compared. Frequency errors between the model and experimental data are used to judge the quality of the update for each case. The frequency errors are shown in Table 1 for the different cases examined. The optimal model parameters are also shown in Table 1 for each case.

For a given parameterization (three or four model parameters), the updated model is found to be dependent on the number of modes considered in the model updating. The quality of the fit is also found to be dependent on the number of modes considered. In general, there is a trade-off between the improved quality of the fit over a wider range of modes and the deterioration of the fit for some of the lower modes. For example, increasing the number of modes from seven to nine results in an improved quality of fit over 13 modes for the three parameter case and over nine modes for the four parameter case. However, the quality of the fit for the torsional mode 3 deteriorates for both cases of three and four model parameters.

Increasing the number of parameters from three to four results in a substantial improvement of the quality of the fit of the modes considered. For the case of seven modes, increasing the number of model parameters results in a considerably improved fit over almost all 13 modes, including the modes 10 to 13 that are not used in the model correlation. For the case of nine modes, no improvement in the quality of the fit is observed for the modes 10 to 13.

The largest variation of the model parameters is observed for the modulus of elasticity of the batten elements to be in the range of 0.395 to 1.03. The smallest variation is observed for the mass at the pulley which is found to be approximately very close to 0.85 its nominal value. This 15% apparent decrease in the model mass accounts for the simplified modelling of the cable and the cable/pulley connection. Incorrect modeling of the preload which was not included as a model parameter also has an effect on the value of the concentrated mass at the pulley. The modulus of elasticity of the longeron members is insensitive to the number of modes used for the case of four parameters. However, for the case of three parameters and for 5 and 7 modes, its optimal value decreases by approximately 10%. The optimal stiffness of the steel cables is approximately 0.95 its nominal value for up to seven modes, and decreases to 0.89-0.85 its nominal when 9 and 11 modes are considered. The variation of the stiffness of the steel cable is less than 10% over all cases considered.

Large variation in the model parameters as well as relatively poor fit observed in Table 1 for different parameterizations and different number and type of modes included in the analysis are due to measurement and modeling errors. Modeling errors include the inaccurate modeling of the structural conections at the major truss nodes, the simplified modeling of the cable/pulley assembly, the variation of the pre-load values that have been measured in the cables, the degree of fixity of the joints, and the translational and rotational fixity of the base of the structure to the granite table/ground.

Tests of the components or sub-structures of the IPEX-II boom are expected to provide a more reliable finite element model for the each component which can be used to built up a more reliable finite element model for the IPEX-II structure. This component-test procedure has been carried out for the Micro-Precision Interferometry (MPI) truss structure and resulted in a significant improved finite element model of that structure [17]. Based on the results of the current analysis, similar tests are recommended to be carried out for the components as well as the sub-structures (bays) of the IPEX-II boom in order to better understand the behavior of the structural joints, the preloaded axial steel cables/pulleys assemblies, and the degree of fixity of the joints and the base of the structure with the granite table. These additional tests will enable us to develop more detailed high fidelity finite element models that will considerably improve the prediction accuracy of the model over the whole frequency range identified by the test data.

4. CONCLUSIONS

The proposed model updating methodology was found to have acceptable performance and accuracy, as well as fast convergence, even for relatively large-size structural models like the one considered in this study. Using measured modal data obtained from test of the IPEX-II boom structure, significant improvements in the updated model of the structure are observed. For the given set of modal test data, the class of finite element model is found to be adequate for predicted the measured modal properties of certain types of modes. However, over the whole frequency range measured in the test, there are modes that could not be adequately matched by the class of models and the parameterization considered. This is due to both measurement and modeling errors. Testing of the IPEX-II at the component or sub-structural level is recommended for improving further the quality of the updated model and the reliability of the model response predictions over the whole range of measured modal frequencies.

The model updating technique has been implemented in matlab to enhance the capabilities of the Integrated Modeling of Optical Systems (IMOS) software package developed at Jet Propulsion Laboratory. Currently, the method is capable of updating the stiffness and mass properties of large complex structures consisting of truss, beam, and plate elements.

5. ACKNOWLEDGEMENT

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6 .EFERENCES

[1] Berman, A. Nonunique Structural System Identification. In *Proc. 7th Int. Modal Analysis Conference*, pp. 355-359, 1989.

[2] Beck, J.L. and Katafygiotis, L.S. Updating Structural Dynamic Models and their Uncertainties - Statistical System Identification. *Journal of Engineering Mechanics, ASCE*, 1998 (in press).

[3] Mottershead, J.E. and Friswell, M.I. Model Updating in Structural Dynamics: A Survey. *Journal of Sound and Vibration*, 167(2), 347-375, 1993.

[4] Beck, J.L. Statistical System Identification of Structures. In *Proc. 5th Int. Conf. on Structural Safety and Reliability, ASCE, II*, pp. 1395-1402, 1989.

[5] Natke, H.G. and Yao, J.T.P., Eds. *Structural Safety Evaluation Based on System Identification Approaches*. Proc. of Workshop at Lam-

brecht/Pfatz. Frieder Vieweg and Sohn, Braunschweig, Germany, 1988.

[6] Katafygiotis, L.S. and Beck, J.L. Updating Structural Dynamic Models and their Uncertainties - Model Identifiability. *Journal of Engineering Mechanics, ASCE*. 1998 (in press).

[7] Stubbs, N., Broome, T.H. and Osegueda, R. Nondestructive Construction Error Detection in Large Space Structures. *AIAA Jornal*, 28(1), 146-152, 1990.

[8] Beck, J.L., Vanik, M.W. and Katafygiotis, L.S. Determination of Stiffness Changes from Modal Parameter Changes for Structural Health Monitoring. In *Proc. First World Conference on Structural Control*. Pasadena, CA, 1994.

[9] Alvin, K.F. Finite Element Model Update via Bayesian Estimation and Minimization of Dynamic Residuals. *AIAA J.*, 35(5), 879-886, 1997.

[10] Farhat, C. and Hemez, F.M. Updating Finite Element Dynamics Models Using an Element-by-Element Sensitivity Methodology. *AIAA Journal*, 31(9), 1702-1711, 1993.

[11] Papadimitriou, C., Levine-West, M. and Milman, M. Structural Damage Detection Using Modal Test Data. In *Proc. Int. Workshop on Structural Health Monitoring*. Stanford, California, 1997.

[12] Papadimitriou, C., Levine-West, M. and Milman, M. A Model Updating Methodology Using Modal Data. In *Proc. 11th VPI/SU Symposium on Structural Dynamics and Control*. Blacksburg, Virginia, 1997 (in press).

[13] Beck, J.L and Vanik, M.W. Structural Model Updating Using Expanded Modeshapes. In *Proc. 11th Engineering Mechanics Conf.*, Y.K. Lin and T.C. Su, eds. ASCE, NY, pp. 152-155, 1996.

[14] Vanik, M.W. and Beck, J.L. A Bayesian Probabilistic Approach to Structural Health Monitoring. In *Proc. Int. Workshop on Structural Health Monitoring*. Stanford, California, 1997.

[15] Levine-West, M.B., Milman, M. and Kissil, A. Modeshape Expansion Techniques for Prediction: Experimental Evaluation. *AIAA Journal*, 34(4), 821-829, 1996.

[16] Levine-West, M.B., Milman, M. and Kissil, A. Modeshape Expansion Techniques for Prediction: Analysis. *AIAA Journal*, 1997 (submitted for publication).

[17] Melody, J.W, Levine-West M. High Fidelity Modeling of Evolutionary Structures in IMOS. In *Proc. First World Conference on Structural Control*. Los Angeles, California, 1994.

[18] Peng, C-Y, Levine-West M. Tsuha, W. *Interferometry Program Experiment IPEX-II Pre-Flight Ground Modal Test and Modal Correlation Report*. JPL D-14809. Jet Propulsion Laboratory, California Institute of Technology, Pasadena, 1997.

[19] Guyan, R.J. Reduction of Stiffness and Mass Matrices. *AIAA Journal*, 3(2), 1965.

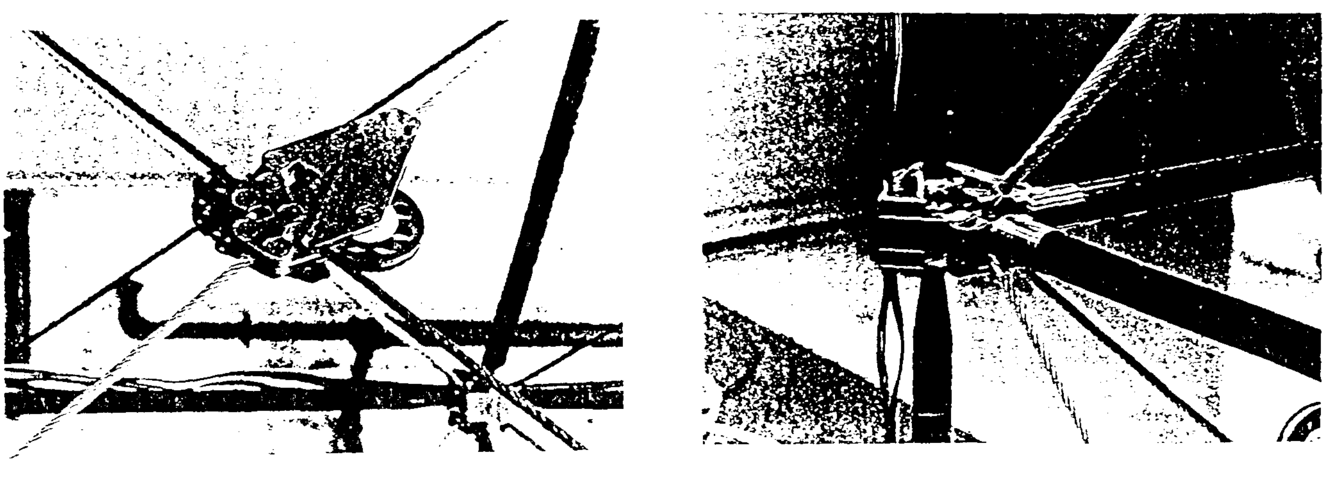

Figure 1. IPEX-II Structure; (a) Steel Cable /Pul ley Connection, (b) Batten/Longeron Joint

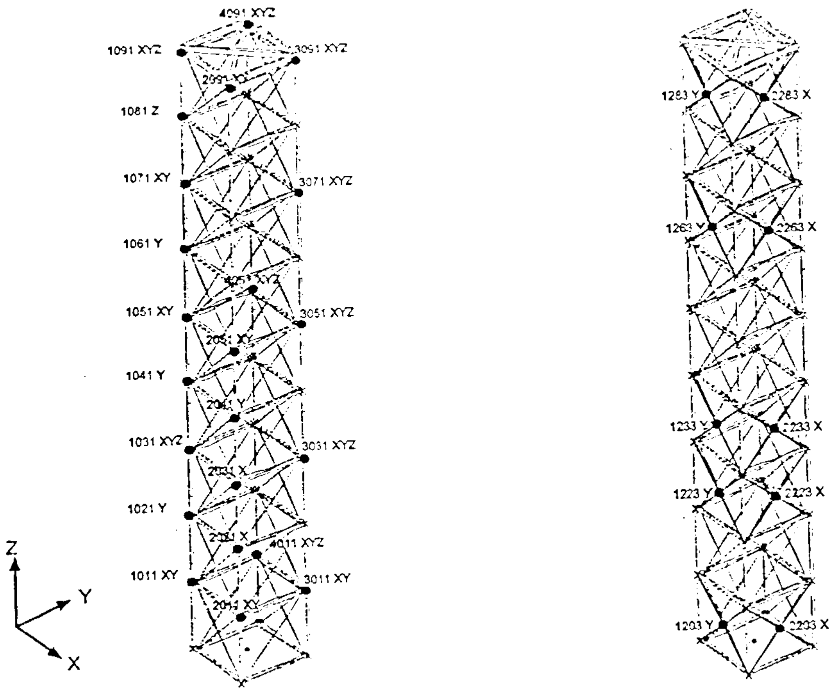

Figure 2: IPEX-II Structure with Accelerometer Locations

Table 1: Results of model updating; Variation in terms of number of model parameters and number of modes included in the analysis

	TEST DAT.4	MODEL 3 P.4R 5 MODES	MODEL 3 PAR 7 MODES	MODEL 3 PAR 9 MODES	MODEL 4 PAR 7 MODES	MODEL 4 PAR 9 MODES	MODEL 4 PAR 11 MODES
Mode # & type	Freq (Hz)	Freq. Err. %	Freq. Err. %	Freq. Err. %	Freq. Err. %	Freq. Err. %	Freq. Err. %
1 (B1)	19. I	0.8	2.4	0.1	0.3	0.4	-0.3
2 (B1)	19.2	0.5	2.0	-0.3	-0.00	0~	-0.7
3 (T1)	32. s	0.2	0.24	-2.9	0.02	-4.1	-4.7
4 (B2+)	71.2	-5,7	-5.0	-6.7	-1.2	-0,6	-2.3
5 (B2+)	71.7	-6.2	-5.6	-7.2	-1.8	-1.3	-2.9
6 (B2-)	105.2	-0.5	1.1	-2.3	0.02	0.3	-0.9
7 (B2-)	105.4	-0.s	0.9	-2.5	-0.2	0.04	-1.1
8 (T2)	10s.9	10:3	10.5	6.7	11.2	6.4	5.5
9 (X1)	136.5	6.5	24.6	0.7	-9.5	0.2	-2.7
10 (B3)	182.5	8.5	11.4	4.9	5.1	5.0	3.6
11 (B3)	183.4	8.0	10. s	4.4	4.6	4.5	3.1
12 (T3)	190.9	9.3	10.7	9.9	4.9	9.8	7.5
13 (X2)	203.3	7.8	9.7	3.5	2.9	5.4	4.5
Parameter Values for the Optimal Model							
θ_1 =		1.15	1.16	1.22	1.25	1.24	1.24
θ_2 =		0.950	0.943	0.890	0.953	0.857	0.848
θ_3 =		0.655	1.03	0.563	0.395	0.525	0.489
θ_4 =		(1.0)	(1.0)	(1.0)	0.855	0.838	0.865